The minus sign corresponds to the minimum of $|\Gamma|^2$ and finally

$$t = x[1 + \{b^{-2}\}],$$

which is the desired result.

When this value of t is substituted in $|\Gamma|^2$, the formula for ρ previously presented results. Of course, the expression for P_L is obtained at the same time.

In the process of differentiation, we have assumed that the length of the resonator, l, is fixed and varied the frequency through the term in λ_g. In a sense, then, we have assumed that the parameters of the coupling elements are fixed with frequency. This however is not an essential assumption. It is well known[12] that the resonant frequency of a lossless waveguide resonator depends only on the length of the waveguide section and not on the frequency behavior of the coupling elements. The effect of introducing loss elements of the order of unity alters the resonant length by the order of b^{-3}. Thus as long as the loss elements vary slowly with frequency, their effect on the resonant frequency will be negligible.

[12] J. Reed, "Low Q microwave filters," Proc. IRE, vol. 38, pp. 793–796; July, 1950.

A General Power Loss Method for Attenuation of Cavities and Waveguides*

J. J. GUSTINCIC†

Summary—**The usual power loss method of evaluating the damping constant and Q of cavities and the attenuation constant of waveguides, as caused by finite wall conductivity, breaks down in the case of degenerate modes and fails to predict the coupling between degenerate modes. By means of variational formulations for the lossy case it is shown how the usual power loss method may be generalized to treat the case when there are degenerate modes present. The generalized method turns out to be a particularly simple extension of the usual procedure.**

THE POWER LOSS technique has always afforded a simple and direct means of calculating the damping and attenuation constants associated with cavities and waveguides having finite wall conductivity. It should be noted, however, that an ordinary power loss analysis is not directly applicable to situations in which a degeneracy between modes is present. As Papadopoulos[1] has shown, degenerate modes are unavoidably coupled together by the surface impedance and thus a single mode approximation no longer gives a sufficient representation of the true fields in the lossy structure. A linear combination of the degenerate modes is then required in the approximation and since the coupling between these modes is not known *a priori*, the power loss technique cannot be applied. Various perturbation solutions have appeared in the literature[1–3] but these solutions fail to give a physical interpretation of the mode coupling and the degree of approximation involved.

Degeneracies are a common occurrence in a large class of geometries and therefore some simplified procedure is highly desirable. It is the purpose of this paper to generalize the usual power loss method so that it is applicable to the degenerate mode case. This generalization is obtained by using the Ritz technique in connection with variational principles for both the cavity and waveguide. The variational approach gives rise to a matrix eigenvalue problem from which all the essential information can easily be obtained. The matrix eigenvalue problems are of the greatest interest and will be presented first while the variational analyses which lead to these conclusions follow to complete the presentation. The following considerations will be limited to the most common situation in which the surface impedance is of the form

$$Z_m = R_m(1 + j),\ R_m \ll Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}},$$

although the analysis can readily be extended, treating a more general form of impedance.

* Received May 14, 1962; revised manuscript received October 19, 1962.

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[1] V. M. Papdopoulos, "Propagation of electromagnetic waves in cylindrical waveguides with imperfectly conducting walls," *Quart. J. Mech. and Appl. Math.*, vol. 7, pp. 325–334; September, 1954.

[2] A. E. Karbowiak, "Theory of imperfect waveguides, the effect of wall impedance," *Proc. IEE (London)*, vol. 102, pt. B, pp. 698–707; 1955.

[3] P. N. Butcher, "A new treatment of lossy periodic waveguides," *Proc. IEE (London)*, vol. 103, pt. B, pp. 301–306; 1956.

The Lossy Cavity

Fig. 1 shows a cavity with inward normal $\boldsymbol{n}$, volume V and surface S. When this cavity has no losses let there be N modes represented by electric and magnetic fields $\boldsymbol{E}_i\boldsymbol{H}_i$, $i=1, 2, \cdots N$, degenerate with resonant frequency ω_0. If losses are now introduced in the walls, the new field should then be approximated by a linear combination of the ideal modes;

$$\mathrm{E} = \sum_{i=1}^{N} a_i \mathrm{E}_i, \qquad \boldsymbol{H} = \sum_{i=1}^{N} a_i \boldsymbol{H}_i, \tag{1}$$

where it is assumed with no loss of generality that the $\boldsymbol{H}_i$ are real vector functions. Associated with the fields of (1) is a time factor $e^{j(\omega_0-\Delta\omega+\sigma)t}$ in which $\Delta\omega$ is the shift in resonant frequency due to losses. Since the fields decay as $e^{-\sigma t}$ the quantity σ is called the damping and the Q of the cavity can be calculated directly by the relation

$$Q = \frac{\omega_0}{2\sigma}\cdot$$

It is desired to obtain expressions for the a_i, σ and $\Delta\omega$. The first step will be to define two sets of matrix elements. A "power loss" element is defined by the integral of two currents over the surface of the cavity

$$P_{ij} = \frac{R_m}{2}\oint_S J_i\cdot J_j dS, \tag{2}$$

$\overline{J}_i$ being the surface current due to the ith mode; $\bar{n}\times\overline{H}_i$. The elements P_{ij} are real and symmetric and in particular, the diagonal elements P_{ii} represent the average power loss exhibited by the ith mode in the usual single mode approximation. In like manner an "energy stored" element is defined as

$$W_{ij} = \frac{\mu_0}{2}\int_V \overline{H}_i\cdot\overline{H}_j dV. \tag{3}$$

Here again W_{ij} is real and symmetric and W_{ii} represents the total average energy stored in the ith mode alone. The generalized power loss approximation then takes the form of the following matrix eigenvalue problem:

$$([P_{ij}] - 2\sigma[W_{ij}])[a_i] = 0. \tag{4}$$

The characteristic equation of (4) will give rise to N values for the damping constant; σ_k, $k=1, 2, \cdots, N$, with corresponding coupling coefficients a_i^k. This result shows that the lossy cavity will possess N distinct modes of the form

$$\overline{E}^k = \sum_{i=1}^{N} a_i^k \overline{E}_i \qquad k = 1, 2, \cdots N, \tag{5}$$

having time factors $e^{j(\omega_0-\Delta\omega_k+j\sigma_k)t}$. As will be shown later, the fact that P_{ij} and W_{ij} are real and symmetric leads directly to the conclusion that the shift in resonant frequency due to losses is just equal to the damping constant,

$$\Delta\omega_k = \sigma_k. \tag{6}$$

It is easily shown[4] that the following orthogonality relation exists between the eigenvectors of (4). If $[a_i^k]$ and $[a_j^r]$ are two distinct solutions of (4), then

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_i^k P_{ij} a_j^r = \sum_{i=1}^{N}\sum_{j=1}^{M} a_i^k W_{ij} a_j^r = 0 \tag{7}$$

for $k\neq r$. Making use of (5), one verifies directly from (7) that

$$\oint_S \overline{J}^k\cdot\overline{J}^r dS = \int_V \overline{H}^k\cdot\overline{H}^r dV = 0, \qquad k\neq r.$$

Thus the modes of the lossy cavity are orthogonal over the volume of the cavity and their surface currents orthogonal over the surface. It is then seen that the linear combinations of degenerate modes chosen to represent the true field will each be in such a proportion that the resulting modes will individually satisfy the usual power loss approximation[5] in which the rate of change of average stored energy is equated to the average power lost in the walls.

$$2\alpha_k\left[\frac{\mu_0}{2}\int_V \overline{H}^k\cdot\overline{H}^k dV\right] = \frac{R_m}{2}\oint_S \overline{J}^k\cdot\overline{J}^k dS.$$

These considerations show the over-all tendency of the degenerate modes to decouple themselves into new fields which do not differ in their properties from the fields resulting from a single mode, nondegenerate, approximation.

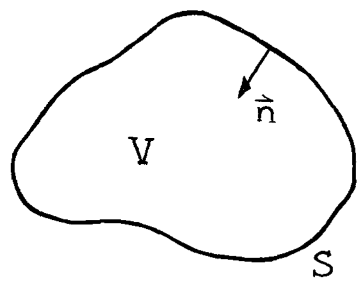

Fig. 1—The general cavity.

The Lossy Waveguide

The geometry of the waveguide is pictured in Fig. 2 where $\bar{n}$, $\bar{\tau}$, and $\bar{a}_z$ are inward normal, tangent, and axial unit vectors respectively. The results for the waveguide now follow analogously to the conclusions for the cavity. The fields of the ideal guide are assumed to be of the form $\overline{H}_n=\bar{h}_{tn}e^{-\gamma_0 z}+\bar{h}_{zn}e^{-\gamma_0 z}$. Here the transverse magnetic field $\bar{h}_{tn}$ is taken as a real vector function and the field of the lossy guide is expanded in terms of

[4] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Co., Inc., New York, N.Y., pp. 570–571; 1960.
[5] See for example, R. Plonsey and R. E. Collin, "Principles and Applications of Electromagnetic Fields," McGraw-Hill Book Co., Inc., New York, N.Y., pp. 368–389; 1961.

N of these ideal modes, degenerate with propagation factor $\gamma_0 = j\beta_0$;

$$\overline{H} = \sum_{k=1}^{N} a_k(\bar{h}_{tk} + \bar{h}_{zk})e^{-\gamma z}. \qquad (9)$$

The propagation factor in the lossy guide is assumed to have the form $\gamma = \alpha + j(\beta_0 + \Delta\beta)$. To find expressions for the attenuation α, the propagation shift $\Delta\beta$ and the coupling coefficients a_n we proceed as before, this time defining a power loss element by

$$P_{ij} = \frac{R_m}{2}\oint_C \overline{J}_i \cdot \overline{J}_j^* dl. \qquad (10)$$

The integration is taken over the perimeter of the guide and $\overline{J}_i$ is the total current due to the ith mode. A "power flow" element is introduced;

$$\mathcal{P}_{ij} = \frac{1}{2}\int_S \bar{a}_z \cdot \bar{e}_{ti} \times \bar{h}_{tj} dS, \qquad (11)$$

where the integration taken over the cross section of the guide. In this case the elements of (10) and (11) are Hermition, *i.e.*, $P_{ji} = P_{ij}^*$ and since $\bar{h}_{tj}$ is real, $\mathcal{P}_{ii}$ represents the average power flowing down the guide in the ith mode. In terms of these quantities the variational analysis requires

$$([P_{ij}] - 2\alpha[\mathcal{P}_{ij}])[a_i] = 0. \qquad (12)$$

Again the matrix eigenvalue problem leads to N distinct modes each with attenuation α_k. The a_i^k and α_k can be calculated from (12) and the fact that the P_{ij} and $\mathcal{P}_{ij}$ are Hermitian will be shown to lead to the conclusion that the losses raise the value of the propagation coefficient by an amount just equal to the attenuation.

$$\Delta\beta_k = \alpha_k.$$

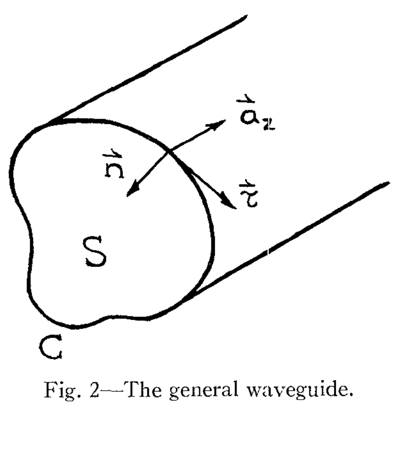

Fig. 2—The general waveguide.

The following orthogonality relations can be shown by employing the procedure used in developing (8)

$$\oint_C \overline{J}^k \cdot \overline{J}^{r*} dl = \int_S \bar{a}_z \cdot \bar{e}_t^k \times \bar{h}_t^r dS = 0 \qquad k \neq r.$$

As before, each new mode will satisfy the power loss approximation independently of the other modes.

VARIATIONAL PROBLEM FOR THE CAVITY

To justify the previous statements by means of the Ritz technique,[6] variational problems corresponding to the boundary value problems of the waveguide and cavity must be found. The cavity problem is easily developed and will be considered first. Employing the usual notation of the calculus of variations, the Helmholtz equation is scalar multiplied by the first variation of the true field and integrated over the volume of the cavity,

$$\int_V [(\nabla \times \nabla \times \overline{H} - k^2\overline{H}) \cdot \delta\overline{H}]dV = 0, \qquad (13)$$

where k is the wave number of the true field; $k = (\omega_0 - \Delta\omega + j\sigma)\sqrt{\mu_0\epsilon_0}$. The first term in (13) can be expanded and converted, in part, to a surface integral,

$$\int_V [(\nabla \times \overline{H}) \cdot (\nabla \times \delta\overline{H}) - k^2(\overline{H} \cdot \delta\overline{H})]dV + \oint_S (\bar{n} \times \delta\overline{H}) \cdot (\nabla \times \overline{H})dS = 0. \qquad (14)$$

On the surface S we have the boundary condition $Z_m(\bar{n} \times \overline{H}) = \overline{E}_{\tan} = jk_0 Y_0(\nabla \times \overline{H})_{\tan}$. Hence (14) can be written

$$\int_V [(\nabla \times \overline{H}) \cdot (\nabla \times \delta\overline{H}) - k^2(\overline{H} \cdot \delta\overline{H})]dV + jk_0 Y_0 Z_m \int_S (\bar{n} \times \delta\overline{H}) \cdot (\bar{n} \times \overline{H})dS = 0. \qquad (15)$$

The left-hand side of (15) is easily recognized as the exact variation of the following quantity:

$$I = \int_V (|\nabla \cdot \overline{H}|^2 - k^2|\overline{H}|^2)dV + jkY_0 Z_m \oint_S |\overline{J}|^2 dS. \qquad (16)$$

The field which causes (16) to be stationary with respect to the first variation in $\overline{H}$ is that field which satisfies the boundary value problem of the lossy cavity. Expansion (1) is now substituted into (16) as an approximation to the true field. This substitution is facilitated by noting the identity

$$\int_V (\nabla \times \overline{H}_i \cdot \nabla \times \overline{H}_j)dV = -k_0^2 Y_0^2 \int_V (\overline{E}_i \cdot \overline{E}_j)dV = \frac{2k_0^2}{\mu_0} W_{ij},$$

which is easily verified by expanding the integral

$$\int_V (\nabla \cdot \overline{E}_i \times \overline{H}_j)dV,$$

[6] Collin, *op. cit.*, pp. 232-247.

and recalling that for the ideal modes, $\bar{n}\times\bar{E}=0$ on S. Using this identity, one finds that I of (16) is proportional to the following sum

$$\sum_{k=1}^{N}\sum_{r=1}^{N} a_k a_r \left[P_{rk} - \frac{(k_0^2 - k^2)}{\mu_0 k_0 Y_0(1-j)} W_{rk}\right]. \tag{17}$$

Now let λ be defined by the expression

$$\lambda = \frac{(k_0^2 - k^2)}{\mu_0 k_0 Y_0(1-j)} \approx \frac{2(\Delta\omega - j\sigma)}{(1-j)}, \tag{18}$$

to the usual order of approximation. Introducing λ into (17) and requiring the variation with respect to the coefficients a_i to vanish, one obtains the matrix eigenvalue problem

$$([P_{ij}] - \lambda[W_{ij}])[a_i] = 0. \tag{19}$$

It is known[7] from matrix algebra theory that an equation such as (19) can have only real eigenvalues so that the λ_k must be real numbers. From (18) it is seen that this can only be true if $\Delta\omega_k = \sigma_k$ and $\lambda_k = 2\sigma_k$. Thus (19) is just the eigenvalue problem (4), previously stated.

Variational Problem for the Waveguide

Although the waveguide presents a two dimensional problem, the formulation of a variational problem is a good deal more complicated than the previous analysis for the cavity. With the field separated into its transverse and axial components the Helmholtz equation separates to yield

$$\nabla_t(\nabla_t\cdot\bar{h}_t) - \nabla_t\times\nabla_t\times\bar{h}_t + k_c^2\bar{h}_t = 0 \tag{20}$$

$$-\nabla_t\times\nabla_t\times\bar{h}_z + k_c^2\bar{h}_z = 0, \tag{21}$$

where ∇_t is the transverse gradient and $k_c^2=\gamma^2+k_0^2$. Maxwell's equations also separate and give the following expressions which will prove useful:

$$h_z = \frac{1}{\gamma}\nabla_t\cdot\bar{h}_t \tag{22}$$

$$\nabla_t\times\bar{h}_t = jk_0Y_0\bar{e}_z \tag{23}$$

$$\nabla_t\times\bar{h}_z = jk_0Y_0\bar{e}_t + \gamma\bar{a}_z\times\bar{h}_t. \tag{24}$$

We begin by scalar multiplying (21) by $\delta\bar{h}_t$ and (22) by $\delta\bar{h}_z$, subtracting the results and integrating over the cross section of the guide to obtain

$$\int_S\{\delta\bar{h}_t\cdot[\nabla_t(\nabla_t\cdot\bar{h}_t) - \nabla_t\times\nabla_t\times\bar{h}_t + k_c^2\bar{h}_t] - \delta\bar{h}_z\cdot[-\nabla_t\times\nabla_t\times\bar{h}_z + k_c^2\bar{h}_z]\}dS = 0.$$

By elementary manipulations this integral can be put into the form

$$\int_S[-(\nabla_t\cdot\bar{h}_t)(\nabla_t\cdot\delta\bar{h}_t) - (\nabla_t\times\delta\bar{h}_t)\cdot(\nabla_t\times\bar{h}_t) + (\nabla_t\times\delta\bar{h}_z)\cdot(\nabla_t\times\bar{h}_z) + (\delta\bar{h}_t\cdot\bar{h}_t - \delta\bar{h}_z\cdot\delta\bar{h}_z)]dS + \oint_C -\bar{n}\cdot[(\nabla_t\cdot\bar{h}_t)\delta\bar{h}_t + \delta\bar{h}_t\times(\nabla_t\times\bar{h}_t) - \delta\bar{h}_z\times(\nabla_t\times\bar{h}_z)]dl = 0. \tag{25}$$

Using (23) and (24) the line integral can be written

$$-\oint_C\left\{\bar{n}\cdot\gamma\left[\left(\frac{1}{\gamma}\nabla_t\cdot\bar{h}_t\right)\delta\bar{h}_t + \bar{h}_t\delta h_z\right] + jk_0Y_0[(\bar{n}\times\delta\bar{h}_t)\cdot\bar{e}_z - (\bar{n}\times\delta\bar{h}_z)\cdot\bar{e}_t\right\}dl. \tag{26}$$

On the perimeter C the boundary conditions are $\bar{e}_z = Z_m(\bar{a}_z\cdot\bar{n}\times\bar{h}_t)$ and $\bar{e}_t = (\bar{\tau}\cdot\bar{n}\times\bar{h}_z)$. With these conditions and (22), (26) can be brought to the form

$$-\oint_C\{\bar{n}\cdot\gamma(h_z\delta\bar{h}_t + \bar{h}_t\delta h_z) - jk_0Y_0Z_m[(\bar{n}\times\bar{h}_s)\cdot(\bar{n}\times\delta\bar{h}_z) - (\bar{a}_z\cdot\bar{n}\times\bar{h}_t)\times(\bar{a}_z\cdot\bar{n}\times\delta\bar{h}_t)]\}dl. \tag{27}$$

Substituting (27) into (25) we find the functional with the desired stationary property;

$$I = \int_S[|\nabla_t\times\bar{h}_z|^2 - |\nabla_t\times\bar{h}_t|^2 - (\nabla_t\cdot\bar{h}_t)^2 + k_c^2(|\bar{h}_t|^2)]dS - jk_0Y_0Z_m\oint_C[|\bar{\tau}\cdot\bar{J}|^2 - |\bar{a}_z\cdot\bar{J}|^2]dl - 2\gamma\oint_V h_z(\bar{n}\cdot\bar{h}_t)dl. \tag{28}$$

Before approximating the true field by expansion (9) a considerable simplification of (28) can be achieved. The approximate field (9) will statisfy the identity

$$\int_s[|\nabla_t\times\bar{h}_z|^2 - |\nabla_t\times\bar{h}_t|^2 - (\nabla_t\cdot\bar{h}_t)^2 + (k_0^2+\gamma_0^2)(|\bar{h}_t|^2 - |\bar{h}_z|^2)]dS = 0. \tag{29}$$

This is most easily shown by replacing $\delta\bar{h}_t$ and δh_z by $\bar{h}_t$ and $\bar{h}_z$, respectively, and setting Z_m and $(\bar{n}\cdot\bar{h}_t)$ to zero in (27) and (25). Also, from (24) it follows that for the ideal modes

$$\int_S|\bar{h}_t|^2 - |\bar{h}_z|^2dS = j\frac{k_0Y_0}{\gamma_0}\int_S\bar{a}_z\cdot\bar{e}_t\times\bar{h}_tdS. \tag{30}$$

Making use of (29) and (30) we have for the approximate fields

[7] *Ibid.*, p. 571.

$$\frac{I}{jk_0Y_0} = \frac{\gamma^2 - \gamma_0^2}{\gamma_0} \int_S \bar{a}_z \cdot \bar{e}_t \times \bar{h}_t dS - Z_m \oint_C \left| \bar{\tau} \cdot \bar{J} \right|^2 - \left| \bar{a}_z \cdot \bar{J} \right|^2 dl. \quad (31)$$

Expansion (9) can now be substituted into (31) and taking note of the fact that if h_{tn} is real then h_{zn} is purely imaginary, (31) becomes proportional to

$$\sum_{k=1}^{N} \sum_{r=1}^{N} a_k a_r \left[\frac{\gamma_0^2 - \gamma^2}{\gamma_0(1+j)} \mathcal{P}_{kr} + P_{kr} \right]. \quad (32)$$

This quantity is of the same form as (16) except here $\mathcal{P}_{kr}$ and P_{kr} are Hermitian. The matrix eigenvalue problem which occurs when the variation of (32) with respect to the a_i is set to zero must also have real eigenvalues. One then finds

$$\frac{\gamma_0^2 - \gamma^2}{\gamma_0(1+j)} \approx -2 \frac{(\Delta\beta - j\alpha)}{(1-j)}$$

must be real, so that $\Delta\beta = \alpha$ and the resulting matrix eigenvalue problem is just (11).

Conclusions

By means of the variational approach, the following general properties of degenerate modes in lossy waveguides and cavities have been shown: 1) The degenerate modes of the ideal structure split into an equal number of nondegenerate modes in the lossy structure. 2) The split occurs such that each of these new modes possess the orthogonality properties of a nondegenerate mode. 3) Each of these new modes individually satisfies the single mode power loss approximation. 4) In the case of the cavity the shift in resonant frequency due to losses is equal to the damping factor and for the waveguide the shift in the propagation factor is equal to the attenuation constant.

In any particular example, the actual calculation of the damping factor or Q of a cavity or the attenuation constant of a waveguide has been systematized into the solving of the determinant of a matrix eigenvalue problem, if desired the field distribution in the lossy structure can also be found by solving for the eigenvectors of the problem, thus obtaining the expansion coefficients of the ideal modes.

Correspondence

Comment on "A Simple Method for Measuring the Phase Shift and Attenuation through Active Microwave Networks"*

In his recent letter[1] on the measurement of phase shift and attenuation of active two ports Alday has submitted material which the author believes should not remain unchallenged. His notion of the addition of power in his system is an erroneous one, so that what he claims to be a measure of attenuation is in fact a measure of a quantity which is not an attenuation at all. It is, as will be seen, a quantity which qualitatively follows the "ups and downs" of a peculiar attenuation.

When the left- and right-hand ports of the network to be measured in Alday's Fig. 1 are designated as 1 and 2, and the network is described by its scattering matrix (S), the wave which reaches the detector of the "attenuator loop" has an amplitude which is proportional to $|S_{11}| + |S_{21}|$ after the indicated phase shifter adjustment. Now, assuming square law detection, one has $|S_{11}|^2 + |S_{21}|^2 + 2|S_{11}S_{21}|$. When the network is lossless (as distinguished by primes) the sum $|S_{11}'|^2 + |S_{21}'|^2$ remains constant at unity; however, $|S_{11}'S_{21}'|$ depends on the details of the network so that Alday's measured "attenuation" is

$$10 \log_{10} \left[\frac{1 + 2|S_{11}'S_{21}'|}{|S_{11}|^2 + |S_{21}|^2 + 2|S_{11}S_{21}|} \right] \text{db}. \quad (1)$$

Assuming $|S_{11}'S_{21}'|$ constant, this quantity, in a vague sense, behaves like

$$10 \log_{10} \left[\frac{1}{|S_{11}|^2 + |S_{21}|^2} \right] \text{db}, \quad (2)$$

where $1/(|S_{11}|^2 + |S_{21}|^2)$ is the ratio of available power (to a matched load) to power *not* dissipated by the network (the sum of the "reflected" and "transmitted" powers). One can only infer that Alday's intention was to measure the peculiar attenuation given by (2) or the related quantity

$$10 \log_{10} \left[\frac{1}{1 - (|S_{11}|^2 + |S_{21}|^2)} \right] \text{db}, \quad (3)$$

which would give, in decibels, the portion of available power dissipated by the two port under test. In any case, Mr. Alday owed it to his readers to define his terms.

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* Received May 22, 1962.

[1] J. R. Adlay, IRE Trans. on Microwave Theory and Techniques (*Correspondence*), vol. MTT-10, p. 143; March, 1962.

Measurement of Impedance at Frequencies above 300 Mc*

I wonder if any of your readers can help me to find out whether the method described below has ever been proposed in the literature to measure the output impedance of signal sources in the frequency range above 300 Mc. Although it is realized that this method is in close relationship with Chipman's method,[1] to the author's best knowledge it has not been proposed for the measurement of active two-terminal impedances, *i.e.*, signal generator output impedances in particular.

The equipment needed is a sliding short-circuit stub with a pickup loop mounted at the short circuit. A quantity proportional to the current in the short circuit is read on a detector which can be any distance (in electrical length) from the pickup loop. If the subscripts of the absolute values of currents denote the corresponding electrical distance of the position of the effective short circuit from the plane where the generator impedance is to be measured, then it can be shown that the generator impedance $Z_g = Z'$

* Received August 29, 1962.

[1] R. A. Chipman, "A resonance-curve method for absolute measurement of impedance at frequencies of the order of 300 Mc/s," *J. Appl. Phys.*, vol. 10; January, 1939.